CONSTRAINTS ON THE πNN COUPLING CONSTANT FROM THE NN SYSTEM[1]

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Abstract

The sensitivity of the deuteron and of pp scattering to the πNN and ρNN coupling constants is investigated systematically. We find that the deuteron can be described about equally well with either *large π and ρ* or *small π and ρ* coupling constants. However, pp scattering clearly *requires* the strong ρ, but favors the weak π (particularly, in $^3P_0$ at low energies). This apparent contradiction between bound-state and scattering can be resolved by either assuming charge-dependent πNN coupling constants or by adding a heavy pion to the NN model. In both cases, the neutral-pion coupling constant is small ($g_{\pi^0}^2/4\pi = 13.5$).

1. INTRODUCTION

Around 1980, it was believed that the πNN coupling constant was well known. Analysing $\pi^\pm p$ data, Koch and Pietarinen [1] obtained $g_{\pi^\pm}^2/4\pi = 14.28 \pm 0.18$ (equivalent to $f_{\pi^\pm}^2 = 0.079 \pm 0.001$). Kroll [2] determined the neutral-pion coupling constant to be $g_{\pi^0}^2/4\pi = 14.52 \pm 0.40$ (equivalent to $f_{\pi^0}^2 = 0.080 \pm 0.002$) from the analysis of pp data by means of forward dispersion relations.

The picture changed substantially in 1987, when the Nijmegen group [3] determined the neutral-pion coupling constant in a partial-wave analysis of pp data and obtained $g_{\pi^0}^2/4\pi = 13.1 \pm 0.1$. Including also the magnetic moment interaction between protons in the analysis, the value shifted to 13.55 ± 0.13 in 1990 [4]. Triggered by these events, Arndt *et al.* [5] reanalysed the $\pi^\pm p$ data to determine the charged-pion coupling constant and obtained $g_{\pi^\pm}^2/4\pi = 13.31 \pm 0.27$. These new values for the πNN coupling constants do not indicate any charge-dependence, but they are considerably smaller (by about 6%) than the determinations of a decade ago. In subsequent work, the Nijmegen group analysed also np data [6] and $\bar p p$ data [7]. In all cases, they find a "small" πNN coupling constant. Their final results are simmarized in Ref. [8]: $g_{\pi^0}^2/4\pi = 13.47 \pm 0.11$ (equivalent to $f_{\pi^0}^2 = 0.0745 \pm 0.0006$) and $g_{\pi^\pm}^2/4\pi = 13.54 \pm 0.05$ (equivalent to $f_{\pi^\pm}^2 = 0.0748 \pm 0.0003$).

In all of their work, the Nijmegen group analyses two-nucleon *scattering* data. The two-nucleon *bound state*—the deuteron—is not considered. On the other hand, the deuteron is very sensitive to the pion. Due to the large radius, the long-range interaction provided by one-pion-exchange (OPE) is dominant in the deuteron [9]. In particular, the quadrupole moment of the deuteron (Q_d) and the asymptotic D-state over S-state ratio (A_D/A_S) are direct consequences of the tensor component of the nuclear force created mainly by the pion. These deuteron properties are known with an uncertainty of less than 2%. Modern realistic

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Table 1. Important coupling constants and the predictions for the deuteron and pp scattering for six models considered in this investigation. The roman numerals in the head of the table refer to the different models explained in the text. Note that Model I–III are wrong either in the deuteron or in pp scattering, while Model IV–VI describe the deuteron *and* pp scattering fair or even well.

	I	II	III	IV	V	VI	Experiment
			Important coupling constants				
$g^2_{\pi^0}/4\pi$	14.4	13.5	13.5	14.0	13.5	13.5	
$g^2_{\pi^\pm}/4\pi$	14.4	13.5	13.5	14.0	14.4	13.5	
$g^2_{\pi'}/4\pi$	0.0	0.0	0.0	0.0	0.0	35.0	
κ_ρ	6.1	6.1	3.7	6.1	6.1	6.3	
			The deuteron				
Q_d (fm^2)	0.278	0.266	0.274	0.273	0.275	0.273	0.276(3)[a]
A_D/A_S	0.0264	0.0251	0.0257	0.0259	0.0261	0.0256	0.0256(4)[b]
P_D (%)	4.99	4.56	5.31	4.75	4.91	5.29	
			$^3P_0$ pp **phase shifts** (deg)				
10 MeV	4.014	3.731	4.105	3.883	3.731	3.727	3.729(17)[c]
25 MeV	9.254	8.612	9.968	8.952	8.612	8.607	8.575(53)[c]
50 MeV	12.39	11.57	14.40	12.00	11.57	11.59	11.47(9)[c]

[a]) Corrected for meson-exchange currents and relativity.
[b]) Ref. [12].
[c]) Nijmegen pp multi-energy phase shift analysis [13].

models for the NN interaction (which all use the old "large" value for the πNN coupling constant, namely, $g^2_\pi/4\pi \approx 14.4$) reproduce, in general, the deuteron properties within the empirical uncertainty of 2%. Therefore, a substantial reduction of $g^2_\pi/4\pi$ (by about 6%) must immediately raise the question whether we can still explain the deuteron.

2. THE DEUTERON

To answer this question, we undertake the following steps. We start from the Bonn (B) potential [10], which we call here **Model I**. This model uses for the pion the old "large" coupling constant $g^2_\pi/4\pi = 14.4$ and for the ρ-meson the large vector-to-tensor ratio $\kappa_\rho = 6.1$ [11]. The predictions by this model for the crucial deuteron quantities are: quadrupole moment $Q_d = 0.278$ fm^2 and asymptotic D-state over S-state ratio $A_D/A_S = 0.0264$ (cf. Table 1). Note that our predictions for Q_d are based on the nonrelativistic impulse approximation and do not include meson-current and relativistic corrections. Therefore, to make the comparison with the experimental data meaningful, we have subtracted from the experimental value for Q_d [0.2859(3) fm^2 [14]] the meson-exchange current and relativistic contributions, which are 0.010 fm^2 for the Bonn potential according to the most recent and very thorough calculation by Henning [15]. Thus, we list 0.276(3) fm^2 in the last column of Table 1 as the empirical quadrupole moment where the assigned error of 0.003 fm^2 is the uncertainty which we assume for the evaluation of the theoretical corrections. Obviously, Model I (Bonn-B) reproduces the empirical quadrupole moment quite well.

In the next step, we lower the pion coupling constant to $g^2_\pi/4\pi = 13.5$, the value suggested by the Nijmegen analysis [8]: we denote this by **Model II** (cf. Table 1). All other parameters

that are crucial for the present discussion are the same as in Model I. (We note that a comparison of two models makes only sense if the deuteron binding energy and the triplet effective range parameters are reproduced accurately and if the over-all fit of the NN phase shifts is satisfactory. To guarantee this, the parameters of the σ-boson and some short-range parameters are always slightly readjusted in the development of the various models discussed in this study.) With this "small" pion coupling constant (and $\kappa_\rho = 6.1$) we obtain $Q_d = 0.266$ fm^2, which is substantially too low.

We now also lower the ρ-meson coupling to $\kappa_\rho = 3.7$ (the so-called vector-meson dominance model value [16]), which leads us to **Model III**. The quadrupole moment now goes up to 0.274 fm^2, which agrees well with the empirical value. Thus, by *lowering both the π and ρ* coupling constants we are able to reproduce the deuteron properties with about the same quality as with *both large π and ρ* coupling constants.

Thus, no clear-cut decision concerning the pion coupling emerges from the deuteron (if we assume the ρ-coupling to be uncertain). We note that we have investigated the deuteron also using other meson models for the NN interaction and have arrived at the same conclusions [17].

3. PROTON-PROTON SCATTERING

The natural next step is to turn to NN scattering. Due to the wealth of partial waves and observables, one would expect to extract more distinctive information from scattering. One question of particular interest is: Is NN scattering equally indifferent towards the choice strong-π+strong-ρ *versus* weak-π+weak-ρ as the deuteron or does it clearly favor one of the two combinations? If one combination is preferred, that could yield the decision the deuteron cannot make. More specifically, one wants to know: Are there clear indications in NN scattering that the small pion coupling constant is to be preferred (as found in the Nijmegen analysis)? And: Does NN scattering clearly favor one of the two ρ couplings?

Before we address these issues, we like to raise a slightly different question, namely: Are there any persistent problems in describing NN scattering (below about 300 MeV laboratory energy) by meson models? Many meson-exchange models for the NN interaction have appeared in the literature over the past 30 years, none of which is perfect (see, e. g., Fig. 5.10 of Ref. [10] for an overview of how modern meson-theory based NN models reproduce the NN phase shifts). In fact, one can identify several typical problems which some meson models have. However, most of these problems typically occur at higher energies (200–300 MeV lab. energy) and, in all of these cases, a solution of the problem is known (and one can always find a counter example that does not have the particular problem). Since the high energy region is sensitive to the short-range part of the NN interaction, the solution of these problems comes typically from heavy-boson exchange, multi-meson exchange, and the parametrization of the meson-nucleon form factors.

Now, there is one single problem that occurs at low energies: essentially all meson models overpredict the $^3P_0$ phase shifts below 100 MeV, *persistently*. We demonstrate this is Fig. 1(a) where the solid line represents the prediction for the $^3P_0$ pp phase shift by Model I. We note that the predictions by other meson models which use the large pion coupling constant are very similar. Since this is a problem at low energies, heavy-boson exchange, multi-meson exchange, and form factors cannot cure this problem. In fact, there is only one single way to solve this problem: *A smaller πNN coupling constant.* The dashed line in Fig. 1(a) shows

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the result when the small coupling constant $g_\pi^2/4\pi = 13.5$ (Model II) is used; numerical values for the $^3P_0$ pp phase shifts for Model I ($g_\pi^2/4\pi = 14.4$) and Model II ($g_\pi^2/4\pi = 13.5$) are listed in the lower part of Table 1. It is seen that this small change in the pion coupling constant has a very large effect on the $^3P_0$ phase shift, and for the smaller coupling constant (Model II) there is excellent agreement with the phase shift analysis. Other partial waves are little affected by this change of the πNN coupling constant, and as far as there is a (small) unwanted effect, it can always be counterbalanced by very small changes in some of the other meson parameters. In summary, the problem of predicting persistently too large $^3P_0$ phase shifts at low energies can only be solved by introducing a "small" πNN coupling constant, a measure against which other partial waves are essentially indifferent.

Unfortunately, this does not finish the discussion; in fact, the real problem starts now. NN bound state and NN scattering must, of course, be explained by one and the same model for the NN interaction. Now, from our previous discussion we know that the deuteron can tolerate a small pion coupling constant only if also the ρ-meson coupling is weak. So when pp scattering needs the weak pion coupling, then—because of the deuteron—we also have to lower the ρ coupling, i. e., we have to use Model III (weak-π+weak-ρ). The pp phase shifts predicted by this model are shown by the dotted line in Fig. 1(a) and Fig. 2 for $^3P_0$ and $^3P_2$, respectively. Clearly, when using $\kappa_\rho = 3.7$, the phase shifts of $^3P_0$ and $^3P_2$ come out totally wrong. Thus, pp scattering requires by all means the strong ρ, but favors the weak π, a combination excluded by the deuteron. We have a problem here.

4. SATISFYING pp SCATTERING AND THE DEUTERON
Presently, we see three possibilities for a solution of the problem:

1. One may try a *compromise* between the large and small pion coupling constant, e. g., $g_\pi^2/4\pi = 14.0$; we denote this by **Model IV**. The deuteron quadrupole moment then comes out at the lower limit (cf. Table 1), while the $^3P_0$ pp phase shifts are already moderately too large [Fig. 1(b)]. It is questionable if one may consider this model as satisfactory.

2. One may assume *charge-dependence of the pion coupling constant*. Using $g_0^2/4\pi = 13.5$ for the neutral pion and $g_{\pi\pm}^2 = 14.4$ for the charged pion defines our **Model V**. $^3P_0$ pp is identical and as excellent as in Model II, but now the deuteron quadrupole moment is improved to 0.275 fm^2, due to the larger $g_{\pi\pm}$. Both pp scattering and the deuteron are described well [cf. Table 1 and Fig. 1(b)].

3. Following the suggestion of Ref. [18], one may include the exchange of a heavy pion of mass 1200 MeV, $\pi'(1200)$, in the meson model for the NN interaction, which is done in our **Model VI**. The additional tensor force provided by π' improves the deuteron just satisfactorily, while it does not deteriorate the $^3P_0$ pp phase shifts.

5. CONCLUSIONS
NN scattering requires by all means a strong ρ, consistent with the determination by Höhler and Pietarinen [11]. There are clear indications in pp scattering to favor a small neutral-pion coupling constant of $g_{\pi^0}^2/4\pi \approx 13.5$. The charged-pion coupling constant cannot be pinned

down from scattering data with similar precision, since the np data are typically of lower quality than the pp data. However, the deuteron requires a large (charged-)pion coupling constant unless new tensor-force generating mechanisms are introduced.

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FIGURE CAPTIONS

Figure 1. $^3P_0$ phase shifts of proton-proton scattering. In part **(a)**, the predictions by Model I (solid line), II (dashed), and III (dotted) are shown; while in part **(b)**, the predictions by Model IV (solid line), V (dashed), and VI (dotted) are displayed. The solid dots represent the Nijmegen pp multi-energy phase shift analysis [13].

Figure 2. $^3P_2$ phase shifts of proton-proton scattering. The predictions by Model I (solid line), II (dashed), and III (dotted) are shown. The solid dots represent the Nijmegen pp multi-energy phase shift analysis [13].

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